Efficient Compilation to Event-Driven Task Programs

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January 22, 2016

Abstract

As illustrated by the emergence of a class of new languages and runtimes, it is expected that a large portion of the programs to run on extreme scale computers will need to be written as graphs of event-driven tasks (EDTs). EDT runtime systems, which schedule such collections of tasks, enable more concurrency than traditional runtimes by reducing the amount of inter-task synchronization, improving dynamic load balancing and making more operations asynchronous.

We present an efficient technique to generate such task graphs from a polyhedral representation of a program, both in terms of compilation time and asymptotic execution time. Task dependences become materialized in different forms, depending upon the synchronization model available with the targeted runtime.

We explore the different ways of programming EDTs using each synchronization model, and identify important sources of overhead associated with them. We evaluate these programming schemes according to the cost they entail in terms of sequential start-up, in-flight task management, space used for synchronization objects, and garbage collection of these objects.

While our implementation and evaluation take place in a polyhedral compiler, the presented overhead cost analysis is useful in the more general context of automatic code generation.

1 Problem and Context

The race for hardware speed and low-power is bringing computers from embedded to large scale into the “Extreme scale” era, in which high numbers of cores react heterogeneously to their environment, and are constrained by their global energy consumption. This imposes tall requirements on the software, which must be as parallel as possible to take advantage of the cores, and also adaptable to changing core capabilities and avoid wasting energy.

One way to address this problem is to depart from the Bulk-Synchronous Programming (BSP) model. Ironically, while BSP has historically promoted parallelism by enabling simple programming models such as loop parallelism and Single-Program Multiple Data (SPMD) computations, the model now seems to stand in the way of the amounts of parallelism sought out. First, bulk synchronizations (across iterations of a for loop, for instance) often express an over-approximation of the actual dependences among computation instances (whether they are tasks or loop iterations). Also, synchrony often results in a loss of parallelism and a waste of energy, since cores spend a portion of their time waiting for some condition to happen (e.g., a barrier to be reached by other cores, a spawned task to return).

Thus, it is commonly believed that the next generation of parallel software will be asynchronous and non-bulk. In other words, programs will be expressed as a graph of tasks, in which tasks are sent for asynchronous execution ("scheduled"), and they become runnable whenever their input data is ready. In this model, the more accurate the inter-task dependences are with respect to the semantics of the program, the more parallelism is exposed.

Some programming models support the expression of parallel programs as recursive tasks, with for instance Cilk [4], X10 [7], and Habanero [6]. In these models, each task can only depend on one (parent) task, or on the set of tasks scheduled by a sequential predecessor. A major advantage of these models is that they offer provable non-deadlock guarantees. However, this comes at the cost of being less general than other systems which express programs as acyclic graphs of tasks [16, 13, 12, 14, 3, 5, 30][1]. While these graphs exist in the literature under a variety of names, here we are using the name "Event-Driven Tasks" (EDT) to refer to them. An event here represents the satisfaction of a dependence.

Since with more generality and performance also come higher programming difficulty, we have worked on tools to automatically *generate* such programs, when they can be modeled with the polyhedral model. With this model, geared towards compute-intensive loop codes, the parallelization tool is provided with a precise representation of the program, whose task graph can be generated statically, as described in [2, 14].

Since dependences are determined statically at parallelization time, relying on systems that discover dependences at runtime [3, 5] would be wasteful and is hence not considered in this paper.

One of the intrinsic challenges of automatic parallelization is to define programmatic ways of generating tasks and dependences and of using the target system capabilities without introducing too much overhead. Another important challenge with polyhedral representations is to maintain optimization time tractable, which requires the use of nimble operations on polyhedra representing the tasks and their dependences.

[1]Since dependences represent constraints on the task exectution order, a task graph needs to be acyclic for a valid execution order of its tasks to exist.

This paper offers two main contributions related to the automatic generation of EDT codes, and in particular the dependence relationships among tasks.

After comparing run-time overheads implied by implementation strategies based on a set of basic synchronization models available in current EDT runtimes, we propose a nearly-optimal strategy based on a slight improvement of one of the models, in section 2.

Then, focusing on the case of the polyhedral model, we present a novel, scalable technique for automatically generating tasks and dependences in section 3.

In section 4, we show how this model can be used to generate EDT codes with the discussed synchronization models, along with further code optimizations. Finally, we evaluate the benefits of using our techniques on a set of benchmarks in section 5, discuss related work in section 6 and summarize our findings in section 7.

2 A comparison of synchronization models

Throughout this paper, we care about the automatic, optimal generation of EDT-based codes, and the cost of using various synchronization models. We are excluding other questions such as the per-task overhead of the runtime, which boil down to the constraint of making the tasks large enough (thousands to tens of thousands of operations per task seems to be the norm on x86-based platforms).

Expressing a program as a graph of event-driven tasks (EDTs) requires some amount of bookkeeping. We are interested in overheads that such bookkeeping would entail, and in their behavior as the number of tasks grows. We illustrate these overheads by referring to the system proposed by Baskaran et al [2]. While we remind the reader that its authors did *not* intend it for a large-scale system, it has been used in larger-scale works for automatic parallelization to task graphs using the polyhedral model since then [9, 14].

Let n be the number of vertices in the task graph. One of the advantages of the reference method is its simplicity. A master thread sets up a graph of tasks linked by dependences; then, an on-line list scheduling algorithm defines when and where tasks get executed.

One obvious bookkeeping overhead in this scheme is that it requires a setup phase *before* the program can actually run in parallel. Amdahl's law dictates that the cost of sequential part of bookkeeping tasks must be insignificant as compared to the execution time of a task. The importance of this **sequential start-up overhead** becomes greater as the available parallelism grows, and is hence crucial to minimize.

Also, the spatial cost of representing the dependences has a major impact on scalability, and even feasibility of generating executable programs. For instance, the baseline method represents inter-task dependences explicitly, all at once, and hence has an $O(n^2)$ **spatial overhead**.

Another source of overhead relates to the amount of tasks and dependencies the runtime has to manage at a given point in time. EDT-based runtimes make it possible to run tasks asynchronously, i.e., a task can *schedule* other tasks, even before its inputs are ready and without waiting for its completion. It is the user's responsibility to let the runtime know when a task's inputs are ready (i.e., when the task can be executed) through the synchronization constructs provided by the runtime API. The amount of tasks that are scheduled before they are ready to execute is referred to here as **in-flight task overhead**. The number of unresolved dependence objects that need to be managed by the runtime at any time is the **in-flight dependence overhead**.

Finally, garbage collection of objects created for the purpose of running a task graph may entail large overheads, especially if the garbage collection must be done only after a large set of tasks has completed. Here, we measure **garbage collection overhead** by the number of objects that are not useful anymore but are not destroyed at any point in the execution.

In the next section, we go through the synchronization paradigms we have experimented with, and examine the overheads induced by their use in automatic code generation.

2.1 Synchronization constructs

We have implemented task-graph code generation based on three synchronization models that we observed in existing task-graph-based runtimes. In one of them, a task (often called "prescriber task") sets up input dependences for a task before these dependences can be satisfied by other tasks. Here we call it the *prescribed* synchronization model.

Alternatively, dependence satisfaction information can go through *tags*, objects that tasks can *get* from and *put* into a thread-safe table. At a high level, tags are identified with inter-task dependences, and tasks check that their input dependences are satisfied by *get*ting the corresponding tag. When a task satisfies another task's input dependence, it puts the corresponding tag into the table. The structure of the tasks our tool generates using tags is defined by a sequence of *get*s, then the task's computation, followed by a sequence of *put*s. The tag table mechanism is currently available in the SWift Adaptive Runtime Machine [13] (SWARM) runtime. It has also been proposed for future implementation in the Open Community Runtime [16] (OCR) 1.0.1 specification.

Finally, tasks may be associated with a counter, and scheduled upon the counter reaching zero. In this case, only the number of dependences are represented, as opposed to a set of dependences. A *counted dependence* is a synchronization construct that associates the scheduling of a task with a counter. The task is scheduled whenever the counter reaches zero. We extended this construct to one that safely creates a counted dependence that needs to be decremented if it does not exist yet, and called this construct "autodec."

Table 1 presents a breakdown of the synchronization constructs available in the Exascale-oriented runtimes studied in this paper.

In OpenStream, a main thread sets up "streams", which are both a synchronization and communication queue between tasks. The order in which tasks that write to a stream are sched-

Runtime	Prescribed	Tags	Counted
OCR	×		×
SWARM		×	×

Table 1: Task synchronization models, and examples of runtimes implementing them.

uled by the main task defines which readers of the stream depend on which writers. In that sense, the synchronization model here is akin to a prescribed model. Tags are available in Intel's implementation of the CnC coordination language [12], as well as prescription. The `async` and `finish` constructs available in Cilk, X10 and Habanero can be implemented using counted dependences. Futures can be supported as well, for instance by adding prescription or tags.

The next section studies the asymptotic overheads linked with the use of these synchronization models.

2.2 Comparative overheads

In this section, we go through four synchronization models and find out the implied overheads when using them in automatic code generation. Different ways of using them are considered, when these lead to different overheads. Our analysis is summarized in Table 2, where n is the number of tasks in the graph, and r is the maximum number of tasks that are ready to run simultaneously in any possible execution of the graph. d is the complexity of computing the number of predecessors to a task, and o is the maximum number of dependences going out of a task (the maximum out-degree in the task graph).

The following sections (2.2.1 through 2.2.4) go through a detailed analysis for each synchronization model. They ultimately show that optimal overheads can be obtained by using autodecs.

2.2.1 Prescribed synchronization

To support our discussion, let us consider a particular task in a task graph, and let us refer to it

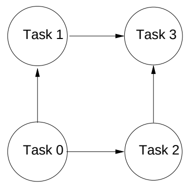


Figure 1: Simple example where a successor task has two predecessors

as the *target task*. We also define n as the number of tasks in the graph to run on the runtime system. For simplicity, we also assume a "master" thread/task/worker, which is able to schedule tasks.

With prescribed synchronization, the target task is created and its input dependences are set up by a task that precedes it in the task graph. This method is straightforward for task graphs that are trees, but less so in cases where tasks may have more than one direct predecessor. To illustrate this, consider the case when a task has more than one predecessor, illustrated with a "diamond" pattern in Figure 1, and in which Task 3 is the target task. In this toy example, dependences do not define a particular order of execution between Tasks 1 and 2. Task 3 needs to be created, and its input dependences set up by one of its predecessors. Without further synchronization, it is impossible for Task 2 to know whether Task 1 has created Task 3 already, or if it needs to be created, and conversely for Task 1. When the target task has more than one predecessor, and without further synchronization possibilities, there are three methods available:

1. A task that dominates all the transitive predecessors of the target task can be responsible for the creation of the target task. The only dominator in our example is Task 0. In our example, Task 0 would indeed be responsible for the setup of all the other tasks.
2. Alternatively, one of the transitive predecessors to the target task is chosen to set up the target task, and additional dependences are

	Start-up	Spatial	In-flight tasks	In-flight deps	Garbage collection
Prescribed	$O(n^2)$	$O(n^2)$	$O(n)$	$O(n^2)$	$O(n)$
Tags Method 1	$O(1)$	$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$
Tags Method 2	$O(1)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Counted	$O(n.d)$	$O(n)$	$O(n)$	$O(n)$	$O(1)$
Autodec w/o src	$O(1)$	$O(n)$	$O(n)$	$O(r.o)$	$O(1)$
Autodec w/ src	$O(1)$	$O(r.o)$	$O(r)$	$O(r.o)$	$O(1)$

Table 2: Overheads associated with task graph synchronization models

introduced between the chosen task and the transitive predecessors to the target task that don't precede the chosen task. The additional dependences materialize the fact that the created target task becomes an input dependence to the transitive predecessors. In our example, we could for instance choose Task 1 as the creator and add a dependence between Tasks 1 and 2, making the task graph entirely sequential.

3. Finally, a special prescriber task can be added for the specific purpose of setting up the target task. Dependences representing the created task object are added between the prescriber task and a set of tasks that transitively precede and collectively dominate the target task. One instance of such set is the set of direct predecessors to the target task.

Since one of the fundamental goals of EDT runtimes is to increase the amount of available parallelism, the loss of parallelism induced by the introduction of sequentializing dependences [2] in Method 2 excludes it from the set of acceptable methods.

The worst case for Method 1 is when the task graph is dominated by one task, as in the diamond example. This occurs fairly frequently, including in many stencil computations as parallelized through polyhedral techniques. In this case, the dominator task is responsible for setting up all tasks in the graph and their dependences, before any other task can start. An equivalent case is when all tasks are dominated by a set of tasks, in which case the host is the only common dominator of all tasks. Both cases result in a $O(n^2)$ sequential overhead, which accounts for the setting up of all the dependences in the graph.

[2] "sequentializing dependences" are dependences that are not required by the semantics of the program and that reduce parallelism.

A naive implementation of Method 3 would generate a prescriber task for each target task in the graph that has more than one predecessor, and introduce dependences between the prescriber task and a dominating set of predecessors to the target task. Notice that this process adds an input dependence to the predecessors, which may have only had one predecessor before adding the prescriber task. These predecessor tasks now fall into the original problem, and themselves require a prescriber task, which must precede the initial prescriber. This results in a transitive construction of prescriber tasks.

In the worst case, the number of such tasks grows as a polynomial of n, $pr(n)$. This is illustrated in Figure 2, where the number of prescriber tasks grows in $O(n^2)$. The number of dependences for these prescriber tasks is then expected to be in $O(pr(n)^2)$.

In the same example, while some tasks may start before all the prescribing tasks have completed, the number of prescriber tasks that need to complete before any single non-prescriber task in the graph equals $\frac{n-1}{2}$, i.e., a $O(n)$ sequential start-up overhead. The complexity of the sequential start-up overhead appears to be a polynomial of degree less than $pr(n)$.

Such an approach would be impractical at large scale, not only because the potentially high com-

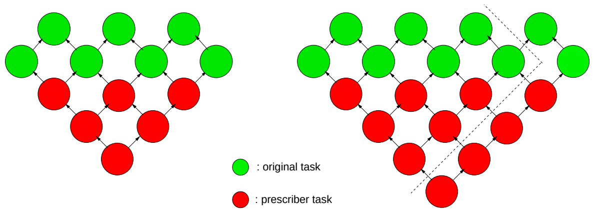


Figure 2: Growth of prescriber tasks as introduced by Method 3.

plexity of its incurred sequential overhead, but also because many additional tasks may need to be created.

A response to both these issues is to group the prescriber tasks into “macro-”’ prescriber tasks, which are responsible for setting up several tasks. Without any particular knowledge of the graph structure, the optimal grouping for these tasks is when creating one single prescriber task that sets up all the graph. This solution is exactly equivalent to Method 1, and it has the same overheads.

Analyzing the graph structure in the hope of finding more than one group with lower overheads also has a $O(n)$ overhead. Hence Method 3 is at best as good as Method 1.

We implemented Method 1 using OCR. Its spatial overhead is $O(n^2)$ because all dependences are represented explicitly. The number of tasks to be handled by the scheduler at once (in-flight task overhead) is $O(n)$, the number of in-flight dependences is $O(n^2)$ and the input dependence objects for each task can be garbage-collected when the task starts.

2.2.2 Tag-based synchronization

Notionally, a tag is a key in an associative table. A predecessor and its successor tasks synchronize through a tag that represents the completion of the predecessor task to the successor task. The predecessor signifies its completion by putting the tag in the table. The successor can wait for tags to be available in the table by getting the tag. Control returns to the task whenever all the tags for which a `get` was issued have been put in the table. To avoid deadlocks, `get`s are typically asynchronous. When a tag is put, all the tasks that did `get` the tag are considered for execution.

We have found two meaningful tag-based methods to perform inter-task synchronization.

1. In the first method, each pair of tasks linked by a dependence is mapped to a tag.
2. In the second method, independently developed in [27], one tag is associated with each predecessor task. Before completion, each task puts a tag that signifies that it has completed. Its successors all get the same tag from the table.

A clear advantage of these methods, as compared to prescribed synchronization methods, is that they have no sequential scheduling overhead, since all tasks can be scheduled in parallel, virtually at any point in time.

The cost of storing synchronization objects (spatial overhead) differentiates both methods. It is $O(n^2)$ for Method 1 (one tag per dependence), and $O(n)$ for Method 2 (one tag per task). A subtlety here is that while only completed tasks perform a `put` in the tag table, all the other tasks may have performed at least one `get`, which needs to be tracked by the runtime. However, Method 1 has an advantage in terms of garbage-collection of its tags, since a tag can be disposed of as soon as its unique `get` has executed. The SWARM runtime offers one-use tags, where the disposal of a tag is performed by the runtime after a `get` was done on the tag. In Method 2, without further sequentializing synchronization, successor tasks don’t know whether they are the last task to `get` the tag. Hence the tag objects can only be disposed once a post-dominator of the task graph has started, or when the entire task graph has completed.

In terms of in-flight task overhead, a straightforward implementation of both methods consists in starting all the tasks upfront and letting them synchronize with each other. To reduce the number of tasks to be managed by the scheduler simultaneously, tasks should be scheduled by their predecessors. The tightest bound on the number of in-flight tasks is obtained when each task is scheduled by one of its predecessors. In this case, the number of in-flight scheduled tasks is exactly the number of tasks that are ready to run. The main obstacle to achieving this appears again with tasks that have more than one predecessor, in which case one of the predecessors must be chosen to schedule the successor task. This cannot be performed dynamically, using tags only, without introducing sequentializing synchronizations.

Methods for statically electing a task within a set of tasks are available in the context of automatic parallelization using the polyhedral model. For any given successor task, the method consists in considering a total order among the set of predecessor tasks, and defining the task that minimizes the order [10, 29] as the one that schedules the successor task. Unfortunately, except for simple cases, the computation of such a minimum doesn't scale well with the number of nested loops in the program, leading to potentially intractable execution times of the automatic parallelization tool. Additionally, this solution is specific to the polyhedral model, and in this section we are discussing synchronization models in the general context of automatic generation of EDT codes.

Hence, here we consider the straightforward solution as the only generally viable option, with a $O(n)$ in-flight task overhead. Method 2, proposed by [27], is superior to Method 1 across the board, except for its garbage collection overhead.

2.2.3 Counted dependences

Counted dependences have similarities with both prescribed synchronizations and tags. Like prescribed synchronization, they require a task to initialize them. A counted dependence naturally represents the number of unsatisfied input dependences of a successor task. It is decremented by each predecessor of the task, at completion. Without further synchronization, counted dependences have an $O(n^2)$ sequential overhead – like prescribed synchronizations – if the input dependences need to be enumerated, or $O(n.d)$ if there exists an analytic function that computes them in time d.

Such a function can be generated in the case of polyhedral code generation. We show in section 4 that task dependences can be represented with a polyhedron, which scans the predecessors (resp. successors) of a task as a function of the task's own runtime parameters. We use polyhedral counting techniques to compute the number of predecessors to a task, either by evaluating the enumerator of the polyhedron [8, 28], or by scanning the polyhedron as a loop and incrementing the count by one for each iteration. The best choice of a counting loop versus an enumerator depends upon the shape of the polyhedron. Complex shapes result in complex enumerators, which can be costlier to evaluate in practice than with a counting loop, especially if the count is low.

The fact that the program starts with n tasks to schedule implies an in-flight task overhead of $O(n)$. Only one counted dependence is required for each task, giving a spatial overhead and an in-flight overhead of $O(n)$. Garbage collection of the counted dependence associated with each task can be performed as soon as the task starts.

The set of useful in-flight scheduled tasks should be the ones that are ready to run, i.e., the ones whose input dependences are satisfied, plus the ones that are already running. Let r be the maximum number of such tasks in any execution of the task graph. Having less than r in-flight tasks would reduce parallelism and is hence not desirable. An ideal task graph runtime scheme would have $O(r)$ in-flight task overhead and, accordingly, an $O(r)$ spatial overhead.

2.2.4 Autodecs

Sequential overhead results from the inability to determine, for a given successor task with multiple predecessors, a unique predecessor that can set up the successor task (let us call such task the successor's creator).

As we saw in previous sections, we assume that there is no general, viable way to resolve this statically. We propose a dynamic resolution based on counted dependences, which does not introduce sequentializing dependences. Again, this is different from dynamic dependence discovery as performed by some runtimes, since the dependences here are defined by the compiler. In our proposed dynamic resolution, the first task to be able to decrement a successor task's counter becomes its unique creator. We call such decrement with automatic creation an `autodec` operation.

Creation of a unique counted dependence – and hence a unique successor task – can be ensured for instance using an atomic operation, which deals with the presumably rare case when two predecessors would complete at the exact same time.

As a result, only tasks that don't have a predecessor need to be scheduled by the master task (no sequential start-up overhead). The tasks that have predecessors are scheduled upon the first completion of one of their predecessors, resulting in an $O(o.r)$ in-flight dependence overhead. Tasks are only scheduled when all their dependences are satisfied, resulting in an $O(r)$ in-flight task overhead.

An $O(r)$ spatial overhead can be obtained by storing the counted dependences in a map (for instance a hash map), at the price of more complex synchronization mechanisms.

Consider the case where the set of tasks without predecessors is unknown statically. Since we know the set of predecessors for each task, one solution would be to identify this set by scanning all the tasks and collecting the ones with zero predecessors. Unfortunately, this would entail a worst-case sequential start-up overhead of $O(n)$, which can be dramatically optimized in the case of the polyhedral model, as presented in section 4.3.

To avoid this need, we introduce a `preschedule` operation, in which a counted dependence is atomically initialized – as in autodecs – but not decremented. The fact that the same mechanism is used by autodec and preschedule operations guarantees that no counted dependence will be created more than once, and that no task will be executed more than once. Hence, the order in which the master task preschedules tasks and tasks auto-decrement their successors does not matter, and preschedule operations can execute concurrently with the tasks, resulting in a $O(1)$ sequential start-up overhead.

Porting autodec principles to the tag-based model: A similar synchronization combined with task initialization could be implemented on top of Tags Method 1, using an "auto-put" operation, through which the first predecessor to a task also sets up the task. Unfortunately, this method would still suffer from a higher spatial overhead ($O(r^2)$), since one tag is associated with each dependence.

It is clear that counting could be used in Tag Method 2 [27] to reduce its garbage collection overhead to $O(r)$. However, we do not see a way around the $O(n)$ overhead for spatial occupancy and number of in-flight tasks.

3 Scalable task dependence generation

Automatic extraction of task parallelism is an attractive proposition. Unfortunately, existing techniques based on the polyhedral model weaken this proposition, because their practicality is limited by their poor algorithmic tractability.

For the sake of simplicity, in this section we assume that each tile defines a task, and we use both words interchangeably. In practice, a task is defined either as a tile or as a set of tiles. Also, a useful guideline is that no synchronization should happen inside a task, which enables the scheduler to prevent any active wait.

The base technique used by [2, 9, 14] to compute tiles and tile dependences is as follows. The authors form dependence relationships among pairs of tiled references. The dependence domain is expressed in the Cartesian product of the tiled iteration domains of the source and destination (polyhedral) statements The task dependences are obtained by projecting out the intra-tile dimensions in both source and destination iteration spaces. In [14], the transformations are actually expressed in terms of a transformation from the iteration domain to a multi-dimensional time range called the schedule. Useful schedules being bijective functions, descriptions based on the domain and the schedule are equivalent in practice. Here, we choose to use the domain-based description because it is simpler.

Unfortunately, the base technique does not scale well because it relies on the projection of a high-dimensional polyhedron (or of integer-valued points in the polyhedron). Projection is know to scale poorly with the number of dimensions of the source polyhedron. This is true even when the rational relaxation of the source polyhedron is considered, a valid and slightly conservative approximation in the case of dependences.

Here, we present a technical solution which does not require the computation of a high-dimensional dependence domain, and also does not rely on projections. Our technique assumes that iteration space tiling partitions computations into parallelotopes. In current polyhedral parallelization, hyperplanes that define the shape of the parallelotopes are defined by scheduling hyperplanes. Together, they form a schedule, which defines a transformation of the domain. Tiling is then performed along these hyperplanes. Since we are eliding the schedule in this description, parallelotope tiling hence corresponds to applying the transformation defined by the scheduling hyperplanes to the iteration domain, followed by orthogonal tiling. Hence, without loss of generality and for the sake of clarity, we are describing our method assuming orthogonal tiling, where the tiling hyperplanes are defined by canonical vectors of the iteration space.

The main idea is to start with a *pre-tiling* dependence (i.e., among non-tiled iterations), and to derive the inter-tile dependences by expressing the tile iteration spaces using a linear compression of the pre-tiling iteration spaces.

Sets of integer-valued points are represented in the polyhedral model by a rational relaxation, which can be represented compactly as the integer points of a (rational) polyhedron. We first explain our technique on a polyhedron D, for which we consider a tiling transformation defined by a matrix G. We show how to precisely define the set of tile indices that correspond to tiles that contain integer points in D. More specifically, let the integer diagonal matrix with positive diagonal elements $G \in \mathbb{Z}^{n \times n}$ represent the orthogonal tiling transformation being applied to the space of index $I \in \mathbb{Z}^n$ in which D is immersed.

The relationship between an iteration I and the inter-tile $T \in \mathbb{Z}^n$ and intra-tile $X \in \mathbb{Z}^n$ dimensions obtained by tiling I according to G is:

$$I = GT + X \tag{1}$$

$$0 \leq X \leq diag(G) - \vec{1} \tag{2}$$

, where $diag(G)$ is the vector made of the diagonal elements of G, and $\vec{1}$ is a n-vector of coefficients 1.

G being invertible, (1) can also be written as:

$$T = G^{-1}I - G^{-1}X, \tag{3}$$

And (2) can be written as:

$$0 \leq G^{-1}X \leq \frac{diag(G) - \vec{1}}{diag(G)}$$

where elementwise division is used.

Let U be defined as:

$$\{Y \in \mathbb{Q}^n : Y = -G^{-1}X, 0 \leq X \leq diag(G) - \vec{1}\} \tag{4}$$

From Equation 3, any T corresponding to an integer point I in D is defined by:

$$T = G^{-1}I + Y, Y \in U \tag{5}$$

Hence, the set of values of T corresponding to integer points in D is given by

$$T \in image(D, G^{-1}) \oplus U \tag{6}$$

where $\oplus$ represents the polyhedral direct sum operator. This set is *exact* for any given D and tiling G of D's space.

We can apply the same method for a dependence $\Delta(I_s, I_t)$ linking the iteration spaces I_s and I_t of a source statement s and a target statememnt t. Let us consider tiling G_s for s and tiling G_t for t. The corresponding inter-tile dependence Δ_T is the set of inter-tile indices T_s and T_t that correspond to an integer point (I_s, I_t) in Δ.

We consider the combined compression transformation $G_{s,t}$ which applies G_s to the I_s space and G_t to the I_t space:

$$G_{s,t} = \begin{pmatrix} G_s & 0 \\ 0 & G_t \end{pmatrix}$$

Let X_s and X_t be the intra-tile dimensions defined by $G_{s,t}$. We get a definition of U in the combined source-target space as in Equation 4:

$$-G^{-1}(X_s, X_t)^T \in U_{s,t} \tag{7}$$

Here too, since $G_{s,t}$ is invertible, we define $P = image(\Delta, G_{s,t}^{-1})$ and we have:

$$\Delta_T = P \oplus U_{s,t} \tag{8}$$

We can hence define an exact inter-tile dependence relationship without resorting to forming high-dimensional polyhedra and, more importantly, without having to project any high-dimensional polyhedron. The only operations we have used are a linear, invertible compression, and a polyhedral direct sum.

While already much more scalable (and we will validate this later on), we could still look for an even more scalable solution. In particular, the direct sum of a polyhedron with a hyper-rectangle is that it will results in a polyhedron with many vertices, which could reduce the scalability of further operations. This can be addressed using a cheap, constraints-oriented way of computing a slight over-approximation of this particular type of direct sums, presented in the next section.

3.1 Preventing vertex explosion

The following technique for reducing vertices in the task dependence polyhedron relies on slightly shifting the constraints of P outwards, until the modified P contains all the points of $P \oplus U_{s,t}$. We call this operation an *inflation* of P w.r.t $U_{s,t}$.

As stated above, the U polyhedron defined in (4) can be written (in the T space) as:

$$-\frac{g_i - 1}{g_i} \leq T_i \leq 0$$

U is a hyper-rectangle. Its vertices are defined by vector (g'), where

$$g'_i = \left\{ \begin{array}{l} 0 \text{ or} \\ -\frac{g_i-1}{g_i} \end{array} \right. , i \in [1, n]$$

Consider a constraint of P, written as $aT + b \geq 0$, where b may contain parametric expressions. We are looking for an offset c such that $aT + b + c \geq 0$ contains all the points of $P \oplus U$.

In other words,

$$a(T + (g')) + b + c \geq 0 \Leftrightarrow aT + a(g') + b + c \geq 0$$

This relationship is respected whenever $c \geq -a(g')$. The maximum value for the right-hand side occurs when $g'_i = \frac{g_i-1}{g_i}$ whenever a_i is positive. Hence the maximum required value for c is:

$$c_{max}(a) = \sum_i a_i.g_{a,i}, \text{ where } g_{a,i} = \left\{ \begin{array}{l} \frac{g_i-1}{g_i} \text{ if } a_i > 0 \\ 0 \text{ otherwise} \end{array} \right.$$

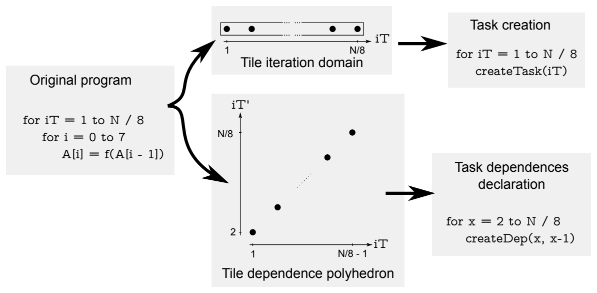
The inflated polyhedron is then defined by replacing the constant offset b with $b + c_{max}(a)$ for every constraint of P. Of course, the U considered for tile dependences is $U_{s,t}$ and the G is $G_{s,t}$. Since the inflated task dependence polyhedron is obtained only by shifting constraints of P, it has the exact same combinatorial structure, i.e., we haven't incresased the number of vertices or constraints through inflation.

4 Generating dependence code

4.1 Prescribed Dependences

In the case of prescribed dependences, the tasks and all their dependences must be declared before starting the program execution. A task is then executed by the runtime layer as soon as all its dependences are satisfied. Polyhedral compilers maintain during the whole compilation the exact set of iterations and dependences as polyhedra, which is translated into the required synchronization APIs.

Task-based parallelization in the polyhedral model relies on forming tasks from tiled loop nest. Here again, for simplicity we assume a task is associated with each instance of a tile, i.e., each value of the inter-tile iterations corresponds to a task instance. Tasks are generated as functions (or their



```
for iT = 1 to N / 8
   for i = 0 to 7
      A[i] = f(A[i - 1])
```

```
for iT = 1 to N / 8
   createTask(iT)
```

```
for x = 2 to N / 8
   createDep(x, x-1)
```

Figure 3: For prescribed dependences, the task creation loop is generated from the tile iteration domain. Task dependences are also declared using the tile dependence polyhedra. The control code generated to handle cases where `N` is not divisible by 8 is not represented to simplify the notation.

equivalent in the targeted runtime), whose parameters include their inter-tile coordinates. Let the "tile iteration domain" of the tiled statements be the set of valid inter-tile iterations, which corresponds to the set of non-empty tasks. The tile iteration domain can be formed by using the compression method of section 3 or by projecting the iteration domain on its inter-tile dimensions. As illustrated in the top of Figure 3, the tile iteration domain is then assigned to a task initialization primitive. A similar tile creation loop nest is created for every distinct tiled loop nest in the program, which results in the initialization code for all the program tasks.

Once tasks are known to the runtime, task dependences are declared. As described in section 3, dependences are formed as a polyhedron in the Cartesian product of the tile iteration spaces of the source and destination tiles. A dependence polyhedron defines a relationship between the inter-tile coordinates of its source tasks and the inter-tile coordinates of its destination tasks. Since, in the prescribed model, the role of such polyhedron is to declare the existence of a dependence, in this section we call it the *declarative dependence polyhedron*. Hence, as explained in [2], they can naturally be generated as loop nests that scan all the (source task, destination task) pairs that are connected by a dependence. A function call is generated as the body of these loops, which declares the existence of the dependence for each such pair, as illustrated in Figure 3. The loop indices are used as the coordinates of the task at the origin and at the destination of the dependence. As shown in our example, the generated loop nest benefits from any of the loop optimizations applied during code generation, including their simplification.

4.2 Tags

It is possible to generate code for Tag Methods 1 and 2 from the declarative form defined above.

In Method 1, each task first gets a tag from each of their predecessors, performs computations, and puts a tag for each of their successors. The get and put loops can be derived directly from the declarative dependence relationship, by mapping the destination inter-tile loops of the dependence polyhedron to the parameters of the task. The task performing the `get`s acts as the destination of the dependence relationship. A loop that scans all the coordinate of the predecessors as a function of the inter-tile parameters of the task is generated from the resulting polyhedron, executing the `get`s. Symmetrically, the iteration domain of the `put` loop is obtained by mapping the source inter-tile dimensions of the declarative dependence to the inter-tile parameters of the task.

Method 2 is simpler in that each task runs a single `put` call, with its own inter-tile parameters as parameters to the `put`. The `get`s are obtained in the same way as for Method 1.

The process is illustrated in Figure 4, where the optimizations performed during code generation simplify the complex loop nest into a single `get` statement parameterized by the task coordinates `iT`.

4.3 Autodecs

Autodecs use counted dependences and atomic task initialization to enable tight dynamic task scheduling. With autodecs, only tasks with no predecessor

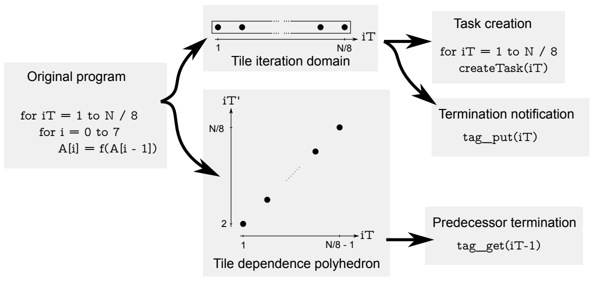


Figure 4: For tags (Method 2), each task issues a put operation for itself and every task waits for its predecessors using a tag get operation. The control code generated to handle cases where N is not divisible by 8 is not represented to simplify the notation.

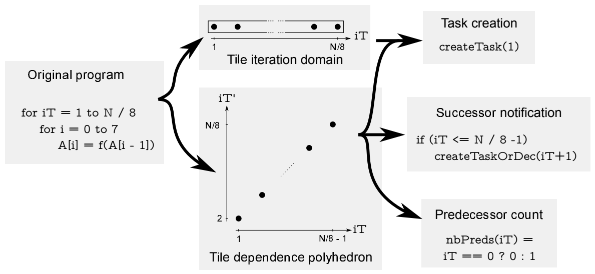


Figure 5: With autodecs, only tasks without predecessors are explicitly initialized. Every task computes its number of predecessors. At completion time, tasks decrement the input dependence counter of their successors, and may initialize them. The control code generated to handle cases where N is not divisible by 8 is not represented to simplify the notation.

need to be created by the master EDT. When a task ends, it iterates over all its successors in order to decrement its number unsatisfied dependences. Such a loop is generated precisely like the put loop in Tag Method 1, except that the function called is autodec instead of put.

The first task which decrements the incoming dependence counter of any of its successors also initializes the successor's counted dependence. To do so, it needs to compute the number of predecessors of said successor task. In order to implement this, a predecessor count function is made available specifically for autodecs by the compiler. This function takes the successor task's inter-tile coordinates and returns the number of its predecessors. The number of predecessors is defined by the number of integer-valued points in the dependence polyhedron, as a function of the successor task's inter-tile coordinates.

There are two possible ways of generating such a function, and both can be defined from the get loop from Tag Method 1. One way is as a loop, by turning the get calls into increments of a counter. The returned value is the number of iterations in the loop, i.e., the number of predecessors to the task. Another way consists in computing the enumerator of the get loop nest, i.e., an analytic function, which returns the number of integer-valued points in the polyhedron representing the get loop, as a function of the inter-tile parameters of the task.

Heuristics to determine which form is best are essentially based on the shape of the polyhedron representing the get loop and an estimate of the number of iterations. Enumerators are not sensitive to the number of iterations, but very much to the shape of the dependence polyhedron, while direct iteration counting is insensitive to shape but undesirable when the number of predecessors is high.

The resulting expression defines the exact number of predecessors for a task as a function of the task coordinates and problem parameters. The generated code is illustrated in Figure 5.

As opposed to the previous methods, with autodecs, the tasks are created by one of their predecessors. A loop scanning the tasks without predecessors has to be created for execution by the master task.

To determine the set of tasks without predecessor, we project the dependence polyhedra on their destination dimensions. The projected polyhedra represent the coordinates of all the tasks with a predecessor. The projected polyhedra are then subtracted from the target statement iteration domain, which results in the set of tasks without predecessors.

5 Experiments

In order to validate our findings, we compare the computation time of our polyhedral dependence computation method with the current state-of-the-art methods [2, 14] in section 5.1. Then, in section 5.2 we explore the question of the significance of the worst-case complexity analysis we presented in section 2 by comparing concrete values for overheads with high worst-case complexity.

“Machine A” is a 12-core, dual-hyperthreaded Xeon E5-2620 running at 2.00GHz with 32Gb RAM running Linux Ubuntu 14.04. Our version of the parallelizing compiler produces source code, which we compile with GCC 4.8.4. “Machine B” is a 32-core, dual-hyperthreaded Xeon E5-4620 running at 2.20Ghz with 128Gb RAM running Ubuntu Linux 14.04.

5.1 Compile-time dependence computation scalability

While it is hardly debatable that performing a linear compressions of low-dimensional polyhedra is (much) less computationally expensive than projecting roughly half the dimensions of a high-dimensional polyhedron, a few well-chosen experiments could help evaluate the importance of the problem.

In order to perform a meaningful comparison, we enforced the same behavior of the polyhedral optimizations upstream (such as affine scheduling and tiling), by running the tool with default options. We also turned off the removal of transitive dependences, so as to leave discussions about trade-offs between compilation time and precision of the dependences out of the scope of this paper. Transitive dependence removal hardly decreases the dimensionality of the problem and increases the number of dependence polyhedra. We instrument the code in order to measure dependence computation time only over 143 benchmarks which include linear algebra, radar and signal processing codes (including FFT-based), stencil computations, sparse tensor codes, an implementation of the Livermore benchmarks [17], and a handful of synthetic codes. The speedups on Machine A are reported on a logarithmic scale in Figure 6.

The high points are as follows.

- Two benchmarks exceeded the 3-minute timeout in the naive method. While this is a global compilation time timeout, the bottleneck there was clearly the computation of the task dependences. These two benchmarks were taken out of the measurements below.

- The average speedup is 10.5X, the maximum (excluding timeouts) is 135X. In order to improve the readability of Figure 6, we arbitrarily capped the timed-out compilations to a 200X speedup. These numbers imply great practical compilation time speedups, considering that these operations are typically the computational bottleneck in the parallelization process. These speedups are relatively low, considering the combinatorial nature of the problem. This is explained by the fact that we are only compiling the code with one level of tiling. If tiling (or any other strip-mining-based transformation) were used to target more than one level of processing or memory, the base number of iteration dimensions would increase significantly, and the gap between our compression method and the projective method would increase dramatically.

- There are a few cases where the projection method is slightly faster than the compression method. We looked at these cases, and the simple explanation there is that the projection is very efficient for these iteration domains.

- Some dependences are computed even for codes that are usually seen as “embarrassingly parallel.” There are two reasons for this. First, code is often partitioned in different ways than with loop parallelism, in order to create more tasks and increase load balancing. For instance, in matrix multiplication, a

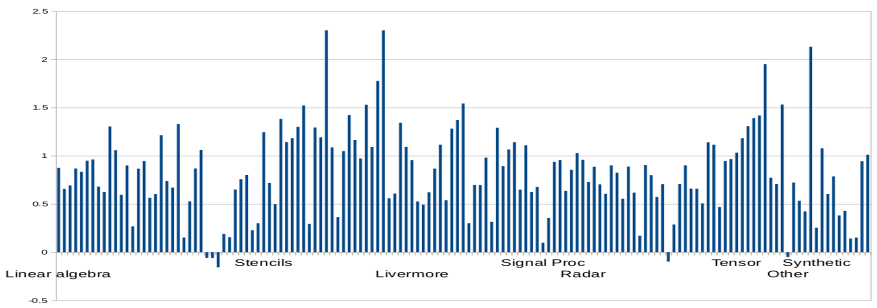


Figure 6: log_{10} of Speedups of the compression method over the projection method

task is created for each tile, i.e., each iteration of the three outer loops (including the reduction loop). Also, we have turned off optimizations that simplify the dependences based on known parallelism information, hence dependences are sometimes built, only to realize later that they are empty.

5.2 Worst-case overheads

The problem of creating meaningful comparisons among codes generated for different synchronization models is somewhat difficult for a few reasons.

First, with OCR we can use prescription and autodecs, while with SWARM we can use tags and autodecs. Comparisons among specific runtimes being out of topic for this paper, we decide to only perform execution time comparisons within each runtime. We compare execution times obtained with prescription and autodecs in OCR, and compare those obtained with tags and autodecs in SWARM.

Similarly, a large space of schedules and tile sizes can be explored in the process of optimizing EDT codes. While they have a great impact on performance and are an interesting topic in and of themselves, they are not quite relevant to the problem we are addressing in this paper. Hence, instead of comparing best execution times among all the possible compiler optimizations we could apply, we chose to pick a particular parallelization choice (the one obtained using the default settings of the compiler), and compare execution times obtained across synchronization models.

The goal of these comparisons is to evaluate the relevance of the worst-case overhead figures we derived in section 2. With Exascale at the horizon, we are easily convinced that they will eventually do, but estimating their impact on current machines is informative of how soon we should start worrying about them.

We do not intend to be exhaustive here but just understand trends, and hence we did not reimplement Tag Method 2 [27], which was neither the optimal nor presenting the most serious overhead behaviors.

We ran a sample set of benchmarks both in prescribed mode and autodec mode using OCR and compared their execution times on machine B. We observed speedups in a majority of the benchmarks (up to 27X for a fixed-point Givens QR code), but roughly a third of them have slowdowns, up to 5X for a synthetic benchmark. Quite systematically, the benchmarks for which the autodec version is slower have short execution times, mostly below 0.1 s, suggesting that they correspond to a small number of tasks.

We also ran a sample set of benchmarks both using Tag Method 1 and autodecs in SWARM. Speedups are more salient there, as autodec-based versions are up to 75X faster (for the `trisolv` benchmark), and one slowdown is observed (10X for `covcol`). This shows that both the synchronization model and the way it is employed should not be overlooked, as they have a major impact on the generated program. Also, three benchmarks did not finish in the tag-based implementation because they ran out of memory, while they do not run out of memory using autodecs. This shows that the $O(n^2)$ spatial cost can be *already limiting* on a 32-core machine.

6 Related Work

The most directly related work targets task-based parallelism from a polyhedral program representation. In particular, Baskaran et al proposed a task-based strategy for multicore processors using the polyhedral model [2]. The strategy is not intended for large scale systems and requires the full set of tasks dependences to be expressed before starting the execution. Our approach has lower memory and computational requirements and is then much more scalable. Moreover, even though tile dependences are considered, they are obtained using polyhedral projection, which is computationally costly. A similar approach is considered more recently by Kong et al [14], whose focus is on the generation of OpenStream code. The same projection-based method is also used in the distributed dataflow work of Dathathri et al[9]. In our method, the tile dependences are deduced from the original program representation, and without requiring any intractable polyhedral projection.

The various proposed solutions focus on different aspects of the polyhedral representation and are often complementary.

An important point about the compression technique is that it addresses one important tractability issue in performing computations on polyhedra during compilation. Tractability is a core problem in polyhedral compilation. It cannot reasonably be ignored in a production compiler. Hence, much work has been performed to improve tractability of polyhedral compilation in the literature, often at the price of approximations or by introducing extra constraints on the program representation. Several techniques restrict the set of constraints allowed to define polyhedra. Several variants of the same techniques exist, each one restricting differently the form of the constraints that can be handled. For instance, Difference Bound Matrices (DBM) only allows constraints in the form $x_i - x_j \leq k, x_i \geq 0, x_j \geq 0$ [23, 20]. Other representations allow more complex constraints such as Unit Two Variables Per Inequality (UTVPI) [1, 21] or Two Variables Per Inequality (TVPI) [24, 25, 26] for instance. The general idea is to restrict the form of the constraints in order to use specialized algorithms to handle the polyhedra, usually with strong worst-case complexity guarantees. In a different direction, Mehta and Yew recently proposed to over-approximate a sequence of statements as a single element called O-molecule [18]. Their approach reduces the number of statements considered in a program, which drastically improves the complexity of several polyhedral operations performed during compilation. A similar solution was proposed by Feautrier [11], and can also be perceived in the work of Kong et al [14]. All the cited improvements are independent from our work and can be combined with the dependence analysis based on tiles presented in this paper.

An alternative, scalable approach to computing tile dependences requires the programmer to express their program in terms of computation (and data) tiles, as in [30].

The second contribution of this paper improves the scalability of the runtime in charge of scheduling the tasks. This is in a context where the runtime is given all dependences by the programmer, as opposed to runtimes that discover dependences as a function of data regions commonly accessed by tasks (as in [3, 15]).

Our proposed improvement does not rely on new language constructs and can be achieved automatically, without involving the programmer. Moreover, the task dependence management we propose is not specifically related to any runtime system, although some of them are better candidates for an integration. We successfully implemented our optimization for two different runtimes: SWARM [13], and OCR [16]. Furthermore, nothing would prevent the implementation of our optimizations on any runtime that enables the composition of programs as a graph of tasks. On the other hand, in some executions models Cilk [4] or X10 [7], the task graph is supported by a tree in which tasks can synchronize with their direct or (respectively) transitive children, which is less general than the model we considered. Such models usually provide termination guarantees in exchange for reduced generality of the task graph model. The polyhedral model provides similar guarantees without specifically imposing tasks trees, although its application domain is more limited than what can be written by hand using tree-supported languages. OpenStream [22] also provides a less restricted task model, although it seems geared towards much finer synchronization granularity based on streams of data.

7 Conclusion

We presented a truly scalable solution for the generation of event-driven task (EDT) graphs from programs in polyhedral representation by a compiler. We investigated tractability issues in both the compilation time and the execution time of such generated programs, and offered two main contributions.

First, we explored the use of three different synchronization models available in current EDT runtimes. We evaluated their overheads in terms of space, in-flight task and dependence management, and garbage collection. We found out a way of using a slight extension of one of the synchronization models to reach near-optimal overheads across the board.

Second, we contributed a method to dramatically reduce the computation time of the costliest operation required to generate EDT codes in a polyhedral compiler: the generation of inter-task dependences. We also discussed how to generate code for the three synchronization models from their polyhedral representation.

Both aspects unlock limitations of EDT code generation for polyhedral compilers. We also believe that our comparative study on synchronization models is useful to anyone who would want to implement an automatic code generation framework based on the ones we considered.

These methods were fully implemented in the R-Stream compiler [19], using the OCR [16] and SWARM [13] runtimes. More optimizations related to inter-task dependences have an impact on performance. Some of them were addressed in the literature, but there are more to be done.

Our polyhedral tile dependence computation method supports most practical tilings out-of-the-box, including diamond tiling. Nevertheless, extensions to more exotic – but useful – tilings, such as hexagonal tiling or overlapped tiling, would be of interest as well.

8 Acknowledgements

This research was developed with funding from the Defense Advanced Research Projects Agency (DARPA). The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressly or implied, of DARPA or the U.S. Government.

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